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# Agent Ontology Alignment Repair through Dynamic Epistemic Logic

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## ABSTRACT

Ontology alignments enable agents to communicate while preserving heterogeneity in their information. Alignments may not be provided as input and should be able to evolve when communication fails or when new information contradicting the alignment is acquired. In the Alignment Repair Game (ARG) this evolution is achieved via adaptation operators. ARG was evaluated experimentally and the experiments showed that agents converge towards successful communication and improve their alignments. However, whether the adaptation operators are formally correct, complete or redundant is still an open question. In this paper, we introduce a formal framework based on Dynamic Epistemic Logic that allows us to answer this question. This framework allows us (1) to express the ontologies and alignments used, (2) to model the ARG adaptation operators through announcements and conservative upgrades and (3) to formally establish the correctness, partial redundancy and incompleteness of the adaptation operators in ARG.

## KEYWORDS

Ontology alignment; alignment repair; agent communication; dynamic epistemic logic

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## 1 INTRODUCTION

Agents use ontologies to represent their knowledge of the world. Generally, these ontologies are not the same. This causes a problem when agents try to communicate: how do the agents understand each other if they express their knowledge in different ways? This question is part of the more general problem of facilitating interoperability between agents while preserving heterogeneity in their information. Ontology matching algorithms have been developed to allow agents with different knowledge representations, structured in *ontologies*, to communicate [22]. These aim to find relationships holding across entities of two ontologies, the *ontology alignment*.

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Alignments are typically computed and provided as input to the agents before any communication or joint task occurs.

Ontology matching algorithms may output only partially correct or incomplete alignments. This means that, even if alignments are available, communication failures can still occur due to mistakes in the alignment. There have been several attempts to repair ontology alignments [1, 17, 21] that have been integrated with multi-agent systems via specific protocols [1, 19]. In these approaches, alignment repair is performed statically, i.e. independently of agent interaction. However, in some multi-agent scenarios, it is not realistic nor desirable for agents to stop interacting until the repair is completed. This is why several approaches to ontology matching have been proposed that attempt to dynamically repair alignments [2, 9, 10]. The Alignment Repair Game (ARG) [11–13] that is inspired by ideas of cultural language evolution [23] is one of them. In ARG, the agents are adaptive: they communicate and, in parallel, evolve alignments through local corrective actions whenever communication fails. This is achieved via adaptation operators that specify precisely how agents adapt the failing correspondence of the alignment.

ARG was evaluated experimentally and the experiments showed that the adaptive agents converge towards successful communication and improve their alignment [11, 13]. However, experiments alone are not sufficient to logically assess properties of operators; whether they are correct, complete or redundant. In this paper we introduce a formal framework based on Dynamic Epistemic Logic [25] to answer this question. This contributes to (1) providing a formal framework for knowledge and belief evolution for logical agents in ARG, (2) formally defining correctness, redundancy and completeness of the adaptation operators, (3) theoretically comparing adaptive agents and logical agents and (4) defining new adaptation operators.

Yet, the scope of this theoretical framework is not limited to ARG: it can be extended to establish formal properties of other games that are designed for agents to improve and repair alignments through interaction. This would allow for a theoretical comparison of different dynamic matching algorithms.

In the remainder, we discuss the related work (§2) and provide the preliminaries (§3). We introduce DEOL (§4) and translate states of ARG to DEOL models (§5). The formal properties of the adaptation operators are then proved (§6). We conclude by emphasizing the contribution to the broader dynamic ontology matching field (§7).

## 2 RELATED WORK

Different techniques have been proposed to evolve alignments: gossiping amongst agents to reach global agreement [1], logical repair to enforce consistency [16, 18, 21] and prevention of logical

violations to agents' ontologies via conservativity principles [17]. These have been integrated with multi-agent systems via specific protocols [1, 19]. However, they are performed independently of agent tasks.

To overcome this problem, *interaction-situated semantic alignment* was proposed [2]. This is an ontology matching algorithm as framed by the interaction protocols used by agents to communicate. Alignments are induced depending on repeated successful interactions and failing interactions lead to revision. This proposal was further advanced to repair alignments *through their use* and generalized to less constrained protocols [9, 10].

The Alignment Repair Game (ARG) [11] is inspired by cultural language evolution [23] to repair alignments through their use. Cultural language evolution offers an experimental methodology in which language (or more generally: culture) is shared amongst a population of agents and evolves through local corrective actions whenever communication fails. ARG adapts this methodology to the evolution of ontology alignments and experiments showed that the adaptive agents converge towards successful communication through local corrective actions [11, 13].

The purpose of this paper is to examine the corrective actions performed by adaptive agents in ARG from a logical perspective. We introduce a formal framework based on Dynamic Epistemic Logic (DEL) to compare adaptive agents to logical agents, and we prove the logical limitations of the adaptation operators proposed in [11, 13]. The logic introduced here is an extension of the work on Epistemic Description Logics [4]. DEL has been widely used as a framework to reason about information flow in multi-agents systems and has been applied communication [7, 25], belief revision [6] and agent interaction [24].

### 3 PRELIMINARIES

In this section, we first explain what are ontologies and ontology alignments (§3.1), then we describe the Alignment Repair Game as a way for agents to evolve alignments (§3.2) and give the syntax and semantics of Dynamic Epistemic Logic (DEL) (§3.3). This logic is the basis for the logic introduced later in this paper.

#### 3.1 Ontologies and ontology alignments

An ontology provides a vocabulary of a domain of interest and a specification of the meaning of terms via semantic relations: specialization ( $\sqsubseteq$ ), equivalence ( $\equiv$ ), exclusion ( $\oplus$ ) and membership ( $\in$ ) [15]. Formally, an ontology can be expressed as a knowledge base in Description Logics [3]. The definition we use in this paper is that an ontology is defined as a quintuple  $O = \langle \mathcal{D}, C, \sqsubseteq, \oplus, \in \rangle$  where  $\mathcal{D}$  is a set of objects,  $C$  is a non-empty set of class names,  $\sqsubseteq, \oplus \subseteq C \times C$  are the semantic relations, and  $\in \subseteq \mathcal{D} \times C$  is the membership relation. To give meaning to these relations an interpretation  $I = \langle \Delta, \cdot^I \rangle$  is provided specifying a domain  $\Delta$  and a function  $\cdot^I$  assigning to object names  $o \in \mathcal{D}$  an element in the domain  $\Delta$  and to class names  $C \in C$  a set of elements of  $\Delta$ . We then say that “ $C$  is subsumed by  $D$ ” ( $C \sqsubseteq D$ ) iff  $C^I \subseteq D^I$ , “ $C$  and  $D$  are equivalent” ( $C \equiv D$ ) iff  $C \sqsubseteq D$  and  $D \sqsubseteq C$ , “ $C$  and  $D$  are disjoint” ( $C \oplus D$ ) iff  $C^I \cap D^I = \emptyset$  and “ $o$  is a member of  $C$ ” ( $o \in C$ ) iff  $o^I \in C^I$ . We also write  $O \models C \sqsubseteq D$ ,  $O \models C \equiv D$ ,  $O \models C \oplus D$  and  $O \models C(o)$ , respectively. For two classes  $C, D$  that overlap (i.e.

that are not disjoint) we also write  $C \not\sqsubseteq D$  and in each ontology  $\top$  is the class such that  $\top^I = \Delta$ . From classes  $C, D$ , we also form the classes  $C \sqcup D$ ,  $C \cap D$  and  $\neg C$  that represent the union, intersection and complement of  $C$  (and  $D$ ). In this paper, the *signature* of an ontology is the set of class names  $C \in C$  and object names  $o \in \mathcal{D}$ .

An alignment  $A_{ab}$  between two ontologies  $O_a$  and  $O_b$  with (generally) different signatures is a set of *correspondences* between classes of the two [15]. Formally, such a correspondence is a triple  $\langle C_a, C_b, R \rangle$  where  $C_a$  and  $C_b$  are class names of  $O_a$  and  $O_b$ , respectively, and  $R \in \{\sqsubseteq, \supseteq, \equiv, \oplus\}$  is a relation that is asserted to hold between  $C_a$  and  $C_b$ . We also write  $C_a R C_b$  for  $\langle C_a, C_b, R \rangle$ .

#### 3.2 Alignment Repair Game

The Alignment Repair Game (ARG) is a protocol designed for adaptive agents to evolve alignments between their ontologies through their use [11, 13]. The aim of ARG is to detect and repair mistakes in alignments whenever a communication failure occurs through application of the adaptation operators. The idea is that ultimately, by repeatedly playing ARG, the alignments converge towards better alignments.

*Definition 3.1 (Adaptation Operator).* An adaptation operator provides a strategy for agents to revise the failing correspondence of the alignment. It specifies, given the failure of the correspondence  $\langle C_a, C_b, R \rangle$  with  $R \in \{\sqsubseteq, \supseteq\}$  and failing object  $o$ , what the agents should do. In [11, 13] the following adaptation operators are introduced:

- delete: delete the correspondence from  $A_{ab}$ ;
- replace: replace the correspondence by  $C_a \sqsubseteq C_b$ ;
- add: in addition to replace, add the correspondence  $C_a^{sup} \supseteq C_b$  between  $C_b$  and the immediate superclass  $C_a^{sup}$  of  $C_a$ ;
- addjoin: in addition to replace, add the correspondence  $C_a^{supO} \supseteq C_b$  between  $C_b$  and the lowest superclass  $C_a^{supO}$  of  $C_a$  that is compatible with the object  $o$  (i.e.  $C_a^{supO}(o)$ );
- refine: in addition to replace, add the correspondences  $C_a \sqsubseteq C_b^{sub}$  between  $C_a$  and all the subclasses  $C_b^{sub}$  of  $C_b$  that are not compatible with the object  $o$  (i.e.  $\neg C_b^{sub}(o)$ );
- refadd: addjoin and refine.

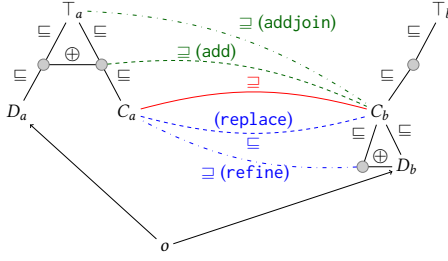
We write  $\alpha_{\langle C_a, C_b, R \rangle(o)}$  for the application of adaptation operator  $\alpha$  to correspondence  $\langle C_a, C_b, R \rangle$  with failing object  $o$ .

From the definition, every operator entails delete, the operators add, addjoin, refine and refadd entail replace and refadd entails addjoin and refine. Furthermore, the order of the actions that are performed by the adaptation operators does not matter. Figure 1 illustrates the effect of the adaptation operators.

*Definition 3.2 (Alignment Repair Game).* The Alignment Repair Game is played by a set of agents  $\mathcal{A}$  with a common set  $\mathcal{D}$  of objects. Each agent  $a \in \mathcal{A}$  is associated with an ontology  $O_a$  and a set  $\{A_{ab}\}$  of non-empty alignments is given between any two ontologies  $O_a$  and  $O_b$  that at least includes  $\top_a \equiv \top_b$ . We write  $O_i \in O_i$  for the most specific class ( $\sqsubseteq$ -wise) of object  $o \in \mathcal{D}$  available in  $O_i$ .

At each round of the game:

- (1) Two agents  $a, b \in \mathcal{A}$  and an object  $o \in \mathcal{D}$  are picked at random.



**Figure 1: Schematic diagram of the deleted (red, solid) and added correspondences (blue and green, dashed and dash dotted) by the different adaptation operators in ARG.**

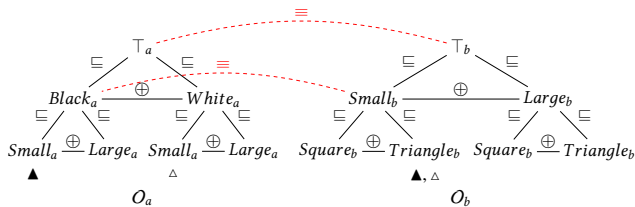
- (2) Agent  $a$  asks agent  $b$  to which class in his ontology the object  $o$  belongs so that it can be translated to agent  $a$ 's ontology via the alignment. Agent  $b$  answers  $C_b$  (with  $O_b \models C_b(o)$  and  $\langle C_a, C_b, R \rangle \in A_{ab}$  where  $R \in \{\sqsubseteq, \equiv\}$ ).
- (3) Agent  $a$  compares  $C_a$  with  $O_a$ . If  $O_a \models O_a \sqsubseteq C_a$ , then the round is a success, else the round is a failure and an adaptation operator  $\alpha_{\langle C_a, C_b, R \rangle(o)}$  is applied to  $A_{ab}$ .

ARG consists of a fixed number of rounds as described above for a chosen operator.

As an illustration of one ARG round consider Example 3.3 that will serve as a running example throughout this paper.

*Example 3.3 (Running Example).* Let agent  $a$  and agent  $b$  play ARG where their ontologies  $O_a$  and  $O_b$  are described in Figure 2. Note that each class in the ontology corresponds to the conjunction of the class label and the label of its ancestors. For instance, the bottom-leftmost class in Figure 2 is defined by  $Square_a \sqcap Small_a$ . The initial alignment  $A_{ab}$  is represented by the dotted red correspondences between classes of their ontologies. Now, consider two cases: the object  $\blacktriangle$  and the object  $\triangle$ . Let in both cases agent  $a$  ask agent  $b$  to which class the object belongs in her ontology so that it can be translated to  $O_a$  via the alignment. In both cases, agent  $b$  will answer  $Small_b$  as both objects belong to this class in  $O_b$ . However, while for the object  $\blacktriangle$  the round would be successful (because  $\blacktriangle \in Black_a$ ), for the object  $\triangle$  a failure is reached (because  $\triangle \in White_a$  and  $White_a \oplus Black_a$ ).

In the latter case  $\alpha_{\langle Black_a, Small_b, \equiv \rangle(\triangle)}$  is applied to the alignment  $A_{ab}$ .



**Figure 2: The ontologies (black) of agent  $a$  (left) and agent  $b$  (right) and the alignment (red, dashed) between them of Example 3.3.**

An ARG state is the state of the alignments reached after a, possibly empty, sequence of rounds where in each failing round an adaptation operator is applied.

*Definition 3.4 (ARG State).* For an ARG game with a set  $\mathcal{A}$  of agents, a set  $\{O_i\}_{i \in \mathcal{A}}$  of ontologies and a set  $\{A_{ij}\}_{i,j \in \mathcal{A}}$  of alignments, an ARG state  $\{A'_{ij}\}_{i,j \in \mathcal{A}}$  is the set of alignments  $A'_{ij}$  reached from  $\{A_{ij}\}_{i,j \in \mathcal{A}}$  after a, possibly empty, sequence of rounds.

For an ARG state  $s$ , we also write  $\alpha_{\langle C_a, C_b, R \rangle(o)}(s)$  for the result of applying the adaptation operator  $\alpha$  to  $s$  with failing correspondence  $C_a R C_b$  and object  $o$ , or simply  $\alpha(s)$  when the correspondence and object are clear from the context.

*Example 3.5 (Running Example).* In case of a failure of the correspondence  $Black_a \equiv Small_b$ , an adaptation operator is applied, adding the following new correspondences to the alignment and deleting the initial correspondence:

- delete: none
- replace:  $Black_a \sqsubseteq Small_b$
- add:  $T_a \sqsubseteq Small_b$
- addjoin:  $T_a \sqsubseteq Small_b$
- refine:  $Black_a \sqsubseteq (Square_b \sqcap Small_b)$
- refadd:  $T_a \sqsubseteq Small_b$  and  $Black_a \sqsubseteq (Square_b \sqcap Small_b)$

By playing ARG with different operators, they can be compared. In Euzenat [11, 13], the operators are compared experimentally in terms of success rate (ratio of successes over rounds played), semantic precision and recall with respect to the known correct reference alignment (the degree of correctness and completeness of the resulting alignment) and convergence (the number of rounds needed to converge). It was found that all the operators have a relatively high success rate, yet do not reach 100% precision, and that recall and convergence both increases with operators that add new correspondences. The operator refadd, followed by add, shows the highest semantic recall and replace, again followed by add, the slowest convergence.

### 3.3 Dynamic Epistemic Logic

Dynamic Epistemic Logics (DEL) are a family of modal logics describing information flow in multi-agent systems. DEL has been widely used as a formal framework to model agent communication [7, 20, 25], belief revision [6] and agent interaction [24]. As such, it provides a solid basis to study knowledge and belief evolution of logical agents playing ARG. Here we consider the syntax and semantics introduced by Baltag, Moss and Solecki [5].

*Definition 3.6 (Syntax of DEL).* The syntax,  $\mathcal{L}_{DEL}$ , of (multi-agent) DEL is defined in the following way:

$$\phi ::= p \mid \phi \wedge \psi \mid \neg \phi \mid K_a \phi \mid B_a \phi \mid [\dagger \phi] \psi$$

where  $p \in \mathcal{P}$  is a proposition,  $K_a$  and  $B_a$  are the knowledge and belief operators for each agent  $a$  and  $\dagger \phi$  with  $\dagger \in \{!, \uparrow\}$  the dynamic upgrades.

The connectives  $\vee$  and  $\rightarrow$ , and the duals  $\hat{K}_a, \hat{B}_a, \langle \dagger \phi \rangle$  are defined in the usual way:  $\phi \vee \psi$  iff  $\neg(\neg \phi \wedge \neg \psi)$ ,  $\phi \rightarrow \psi$  iff  $\neg \phi \vee \psi$ ,  $\hat{K}_a \phi = \neg K_a \neg \phi$ ,  $\hat{B}_a \phi = \neg B_a \neg \phi$ , and  $\langle \dagger \phi \rangle = \neg [\dagger \phi] \neg \psi$ . DEL models are based on Kripke frames with plausibility relations where the logical dynamics act as model transformers.

**Definition 3.7 (DEL Model).** A model of (multi-agent) DEL is a quadruple  $\mathfrak{M} = \langle W, (\geq_a)_{a \in \mathcal{A}}, w^*, V \rangle$  where

- $W$  is a non-empty set of states, or worlds;
- $(\geq_a)_{a \in \mathcal{A}} \subseteq W \times W$  are the plausibility relations on  $W$ , one for each agent, that are converse well-founded, locally connected preorders;
- $w^* \in W$  is the actual world;
- and  $V$  is a propositional valuation mapping propositions to sets of worlds in which that proposition is true.

The plausibility relation  $w \geq_a v$  reads as “ $w$  is at least as plausible as  $v$  for agent  $a$ ” and the epistemic and doxastic relations are defined on  $W$  accordingly:

$$w \sim_a v \text{ iff } w (\leq_a \cup \geq_a) v \quad (1)$$

$$w \rightarrow_a v \text{ iff } v \in \text{Max}_{\leq_a} |w|_a \quad (2)$$

where  $|w|_a$  is the *information cell* (or *accessible cell*) of agent  $a$  at state  $w$  and is defined by:

$$|w|_a = \{v \in W \mid w \sim_a v\} \quad (3)$$

It follows from the properties of  $\leq_a$  and  $\geq_a$  that the relations  $\sim_a$  are reflexive, transitive and symmetric, and the relations  $\rightarrow_a$  are transitive, serial and Euclidean. Therefore they satisfy the usual properties of knowledge and belief, respectively [8, 25].

**Definition 3.8 (Semantics of DEL).** The semantics for DEL is defined in the following way:

$\mathcal{M}, w \models p$	iff $w \in V(p)$
$\mathcal{M}, w \models \phi \wedge \psi$	iff $\mathcal{M}, w \models \phi$ and $\mathcal{M}, w \models \psi$
$\mathcal{M}, w \models \neg\phi$	iff $\mathcal{M}, w \not\models \phi$
$\mathcal{M}, w \models K_a \phi$	iff $\forall v \text{ s.t. } w \sim_a v : \mathcal{M}, v \models \phi$
$\mathcal{M}, w \models B_a \phi$	iff $\forall v \text{ s.t. } w \rightarrow_a v : \mathcal{M}, v \models \phi$
$\mathcal{M}, w \models [\! \phi \!] \psi$	iff $\mathcal{M}^{\downarrow\phi}, w \models \psi$
$\mathcal{M}, w \models [\!\uparrow\phi\!] \psi$	iff $\mathcal{M}^{\uparrow\phi}, w \models \psi$

where  $\downarrow\phi$  and  $\uparrow\phi$  act as model transformers  $\downarrow\phi : \mathcal{M} \rightarrow \mathcal{M}^{\downarrow\phi}$  and  $\uparrow\phi : \mathcal{M} \rightarrow \mathcal{M}^{\uparrow\phi}$  in the following ways, with  $\|\phi\|_{\mathcal{M}} = \{w \in W \mid \mathcal{M}, w \models \phi\}$ :

**Announcement ( $\downarrow\phi$ )** Delete all ‘ $\neg\phi$ ’-worlds from the model.

I.e.  $W^{\downarrow\phi} = \|\phi\|_{\mathcal{M}}, w \geq_a^{\downarrow\phi} v$  iff  $w \geq_a v$  and  $w, v \in W^{\downarrow\phi}$ ,  $V^{\downarrow\phi}(p) = V(p) \cap \|\phi\|_{\mathcal{M}}$  and  $(w^*)^{\downarrow\phi} = w^*$ ;

**Conservative upgrade ( $\uparrow\phi$ )** Change the plausibility orders so that the best ‘ $\phi$ ’-worlds become better than all other worlds, while the old ordering on the rest of the worlds remains. I.e.  $W^{\uparrow\phi} = W, w \geq_a^{\uparrow\phi} v$  iff either  $w \in \text{Max}_{\leq_a} (\|\phi\|_{\mathcal{M}} \cap |w|_a)$  or  $w \geq_a v, V^{\uparrow\phi}(p) = V(p)$  and  $(w^*)^{\uparrow\phi} = w^*$ .

We also write  $\uparrow_1\phi; \uparrow_2\psi$  for the sequence of upgrades  $\uparrow_1\phi$  and then  $\uparrow_2\psi$ . The resulting model  $\mathcal{M}^{\uparrow_1\phi; \uparrow_2\psi}$  is equal to  $(\mathcal{M}^{\uparrow_1\phi})^{\uparrow_2\psi}$ .

The intuition behind the different upgrades is that the trustworthiness of the information source may vary: it may be considered from an infallible source (announcements), or from a trusted, but not infallible source (conservative upgrades). For this reason, conservative upgrades only change the plausibility of worlds without deleting any alternatives.

Note that in all cases,  $w^*$  remains the actual world of the model. This also means that an announcement  $\downarrow\phi$  can only be validly performed on a model  $\mathcal{M}$  if  $\phi$  is true there.

## 4 DYNAMIC EPISTEMIC ONTOLOGY LOGIC

To compare adaptive agents with logical agents, we need a logical framework to model ARG. Here, we introduce Dynamic Epistemic Ontology Logic (DEOL) that is a variant of Dynamic Epistemic Logic where the propositions are object classifications ( $C(x)$ ) and class relations ( $C \equiv D, C \sqsubseteq D$  and  $C \oplus D$ ) of a Description Logic language. This logic enables us to later capture knowledge and belief evolution in alignment repair.

**Definition 4.1 (Syntax of DEOL).** The syntax,  $\mathcal{L}_{\text{DEOL}}$ , of (multi-agent) DEOL is defined in the following way:

$$\phi ::= C(o) \mid CRD \mid \phi \wedge \psi \mid \neg\phi \mid K_a\phi \mid B_a\phi \mid [\!\uparrow\phi\!]\psi$$

$$R \in \{\sqsubseteq, \equiv, \oplus\}, \uparrow \in \{\downarrow, \uparrow\}$$

where  $C, D, \top \in \mathcal{C}, o \in \mathcal{D}, K_a$  and  $B_a$  are the knowledge and belief operators for agent  $a$  and  $\uparrow\phi$  with  $\uparrow \in \{\downarrow, \uparrow\}$  are the dynamic upgrades.

The connectives  $\rightarrow$  and  $\vee$  and the duals  $\hat{K}_a, \hat{B}_a, \langle \uparrow\phi \rangle$  are defined as in the case of DEL.

DEOL models are plausibility models. The difference with DEL models is that instead of a valuation of propositions, we consider a domain of interpretation  $\Delta$  representing the objects and an interpretation function  $I$  assigning to each world a function interpreting each class as a set of objects of the domain.

**Definition 4.2 (DEOL Model).** A model of (multi-agent) DEOL is a quintuple  $\mathfrak{M} = \langle W, (\geq_a)_{a \in \mathcal{A}}, w^*, \Delta, I \rangle$  where

- $W$  is the set of states, or worlds;
- $(\geq_a)_{a \in \mathcal{A}} \subseteq W \times W$  are the plausibility relations on  $W$ , one for each agent, that are converse well-founded, locally connected preorders;
- $w^* \in W$  is the actual world;
- $\Delta$  is the domain of interpretation (a set of objects);
- and  $I$  is an *interpretation function* s.t.  $I(w) = \cdot^{I_w}$  and  $\cdot^{I_w} : \mathcal{C} \rightarrow \mathcal{P}(\Delta)$ , where it holds that  $\top^{I_w} = \Delta$ , and for any two classes  $C, D \in \mathcal{C}$  we have that  $(C \sqcap D)^{I_w} = C^{I_w} \cap D^{I_w}$  and  $(\neg C)^{I_w} = \Delta \setminus C^{I_w}$  for each  $w \in W$ .

We also write  $C \sqcup D$  for the class defined by  $\neg(\neg C \sqcap \neg D)$ , and  $\sqcap\{C_i\}$  and  $\sqcup\{C_i\}$  for the classes defined by  $C_1 \sqcap C_2 \sqcap \dots$  and  $C_1 \sqcup C_2 \sqcup \dots$ , respectively. Their interpretations at world  $w$  are given by  $C^{I_w} \cup D^{I_w}, \bigcap C_i^{I_w}$  and  $\bigcup C_i^{I_w}$ , respectively. In each DEOL model  $\perp = \neg\top$  is the empty class.

The semantics of DEOL is equivalent to that of DEL except that we now have instance classifications  $C(o)$  and class relations  $C \sqsubseteq D, C \equiv D$  and  $C \oplus D$ .

*Definition 4.3 (Semantics of DEOL).* The *semantics* for DEOL extends that of DEL (Definition 3.8) by:

$$\begin{aligned} \mathcal{M}, w \models C(o) & \quad \text{iff } o^{Iw} \in C^{Iw} \\ \mathcal{M}, w \models C \sqsubseteq D & \quad \text{iff } C^{Iw} \subseteq D^{Iw} \\ \mathcal{M}, w \models C \equiv D & \quad \text{iff } C^{Iw} = D^{Iw} \\ \mathcal{M}, w \models C \oplus D & \quad \text{iff } C^{Iw} \cap D^{Iw} = \emptyset \end{aligned}$$

The additional capacities of logical agents compared to the original game are that logical agents can now use the relations between concepts to reason about instance classification. For instance, the following axiom schemata are valid:

$$K_a(C(x)) \wedge K_a(C \sqsubseteq D) \Rightarrow K_a(D(x)) \quad (4)$$

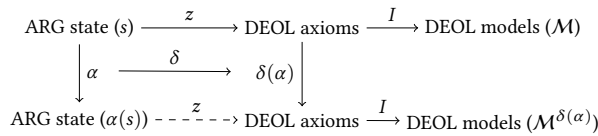
$$K_a(C(x)) \wedge K_a(C \oplus D) \Rightarrow K_a(\neg D(x)) \quad (5)$$

In addition, agents can combine their knowledge and beliefs to obtain new beliefs. For instance,  $K_a(C_a(o))$  and  $B_a(C_a \sqsubseteq C_b)$  entails  $B_a(C_b(o))$ . In other words, agent  $a$  can transfer some of her knowledge about  $C_a$  to beliefs about  $C_b$ .

This increased reasoning capacity of logical agents compared to adaptive agents is crucial in the results later about the correctness, partial redundancy and incompleteness of the adaptation operators.

## 5 TRANSLATION

In the previous section, we have provided a formal framework for knowledge and belief evolution in alignment repair. We now use this framework to capture the Alignment Repair Game. More precisely, we define a translation from ARG states to DEOL axioms that are interpreted as sets of DEOL models (§5.1) and from adaptation operators for ARG to dynamic upgrades on DEOL (§5.2), see Figure 3. These translations are labeled by  $z$  and  $\delta$ , respectively, and the interpretation on DEOL models by  $I$ . This enables us to define and prove correctness, redundancy and completeness of the adaptation operators in the remainder of this paper.



**Figure 3: Diagram of translations from ARG states to DEOL axioms ( $z$ ) that are interpreted by sets of DEOL models ( $I$ ), and from adaptation operators to dynamic upgrades ( $\delta$ ).**

### 5.1 Semantics of ARG states

Let agents  $a$  and  $b$  play ARG with ontologies  $O_a$  and  $O_b$ , respectively, and alignment  $A_{ab}$ . Given the nature of ontologies and alignments, we impose the following *three conditions* on the DEOL axioms describing the epistemic-doxastic states of agents  $a$  and  $b$ :

- Ontology Knowledge (OK)**  $O_a$  ( $O_b$ ) is known to agent  $a$  ( $b$ );
- Alignment Belief (AB)**  $A_{ab}$  is believed by agents  $a$  and  $b$  and;
- Public Signature Awareness (PSA)** The signatures of all ontologies are known to all agents.

In the interpretation on DEOL models, this means that the sentences that describe  $O_a$  are true in any world in  $|w^*|_a$ , and the sentences that describe  $A_{ab}$  are true in all most plausible worlds in both  $|w^*|_a$  and  $|w^*|_b$ .

*Example 5.1 (Running Example).* In the running example, this means that the sentences  $(Square_a \sqcap Small_a) \sqsubseteq Small_a$ ,  $Small_a \sqsubseteq \top_a$ ,  $Triangle_a \sqcap Small_a(\blacktriangle)$ , etc, are true in every accessible world for agent  $a$  and that the sentences  $Small_a \equiv Black_a$  and  $\top_a \equiv \top_b$  are true in the most plausible worlds for both agents.

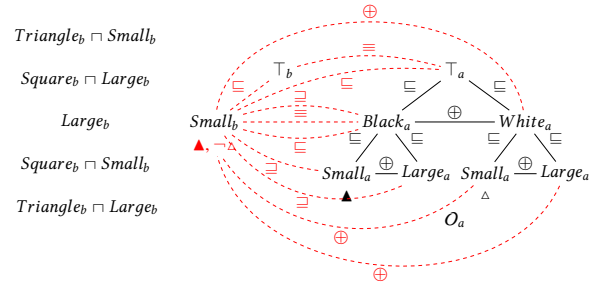
Public signature awareness ensures that agents are allowed to update their information when we consider the dynamics of the adaptation operators. It requires that, for each agent  $a$ , each object  $o \in \mathcal{D}$  and for each two classes  $C, D \in \mathcal{O}_b$  with  $b \neq a$  and not appearing in the alignment, i.e.  $C, D \notin \{C_b \in \mathcal{O}_b \mid \langle C_a, C_b, R \rangle \in A_{ab}, C_a \in \mathcal{O}_a, R \in \{\equiv, \sqsubseteq, \sqsupseteq, \oplus\}\}$ , agent  $a$  considers all combinations of the following alternatives equally plausible:

- $C(o)$  and  $\neg C(o)$
- $C \sqsubseteq D$  and  $C \not\sqsubseteq D$
- $D(o)$  and  $\neg D(o)$
- $C \sqsupseteq D$  and  $C \not\sqsupseteq D$
- $C \equiv D$  and  $C \neq D$
- $C \oplus D$  and  $C \not\oplus D$

Formally, this is achieved on the interpretation on DEOL models by ultimately making as many copies of the worlds describing the agent's knowledge and belief as there are combinations of the alternatives above, ranking them all equally plausible while respecting the order imposed by the alignments.

Because models rapidly explode, we will only draw the information given by the ontologies and alignments.

*Example 5.2 (Running Example).* Figure 4 depicts the epistemic-doxastic state of agent  $a$  at the start of the game. Note that the alignment  $A_{ab}$  consisted of  $\langle \top_a, \top_b \equiv, \rangle$  and  $\langle Black_a, Small_b, \equiv \rangle$ .



**Figure 4: Initial knowledge (solid black lines) and belief (dashed red lines) of agent  $a$  in the Running Example.**

Note that the interpretation of the DEOL translation of ARG with  $O_a$ ,  $O_b$  and  $A_{ab}$  satisfying OK, AB and PSA is not unique. Indeed, there are many variations of models that qualify, and, in particular the *minimal DEOL model*  $\mathcal{M}_{min}^{O_a, O_b, A_{ab}}$  (or  $\mathcal{M}_{min}$  in short when it is clear from the context) in which agents have no other knowledge or beliefs than given by the closure of the three conditions.

**PROPOSITION 5.3.** Any DEOL model  $\mathcal{M}$  describing ARG with  $O_a$ ,  $O_b$  and  $A_{ab}$  that satisfies the three conditions is an extension of the minimal DEOL model  $\mathcal{M}_{min}^{O_a, O_b, A_{ab}}$ .



## 5.2 Dynamics

During the gameplay of ARG, new information is learned. There are two dynamic acts involved in the learning: the communication of  $C_b(o)$  in step 2 of ARG and the adaptation operator applied in step 5 (see Definition 3.2). How do these acts change the knowledge and beliefs of the agents? And are the adaptation operators as defined by Euzenat [11, 13] sufficient to account for these changes?

In order to answer these questions, we translate the communication taking place in ARG to dynamic upgrades on DEOL.

**Definition 5.4 (ARG Dynamics in DEOL).** We model each round of ARG as defined in Definition 3.2 by

$$!C_b(o); \text{ if } \neg C_a(o) \text{ then } \delta(\alpha_{\langle C_a, C_b, \supset \rangle(o)}) \quad (6)$$

where  $\delta(\alpha_{\langle C_a, C_b, \supset \rangle(o)})$  denotes the translation of adaptation operator  $\alpha$  applied to the correspondence  $C_a \supset C_b$  with failing object  $o$ .

Given that  $C_b(o)$  is knowledge to agent  $b$ , the communication of this information in step 3 of ARG translates to an announcement on DEOL. For the adaptation operators, announcements are not the correct tool: adaptation operators tell the agents how to revise the alignment, their beliefs, upon a communication failure. Therefore adaptation operators translate to conservative upgrades.

**Definition 5.5 (Adaptation Operators as Dynamic Upgrades).** Let  $\langle C_a, C_b, \supset \rangle \in A_{ab}$  be the failing correspondence with object  $o$ , the adaptation operators  $(\alpha_{\langle C_a, C_b, \supset \rangle(o)})$  are translated to the following dynamic upgrades on DEOL (where the subscript  $\langle C_a, C_b, \supset \rangle(o)$  is left out for readability):

$$\delta(\text{delete}) = \uparrow(C_a \not\supset C_b)$$

$$\delta(\text{replace}) = \uparrow(C_a \not\supset C_b)$$

$$\delta(\text{add}) = \uparrow(C_a \not\supset C_b \wedge C_a^{\text{sup}} \supset C_b)$$

$$\delta(\text{addjoin}) = \uparrow(C_a \not\supset C_b \wedge C_a^{\text{supO}} \supset C_b)$$

$$\delta(\text{refine}) = \uparrow(C_a \not\supset C_b \wedge \bigwedge \{C_a \supset C_b^{\text{sub}}\})$$

$$\delta(\text{refadd}) = \uparrow(C_a \not\supset C_b \wedge C_a^{\text{supO}} \supset C_b \wedge \bigwedge \{C_a \supset C_b^{\text{sub}}\})$$

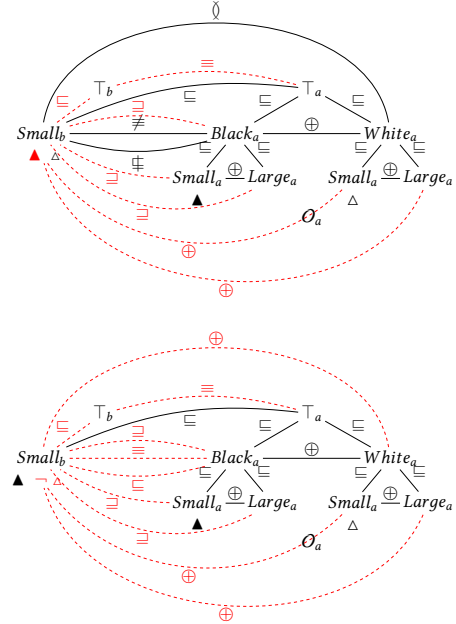
where  $C_a^{\text{sup}} = \text{Min}_{\subseteq} \{C \in \mathcal{O}_a \mid C_a \subseteq C\}$ ,  $C_a^{\text{supO}} = \text{Min}_{\subseteq} \{C \in \mathcal{O}_a \mid C_a \subseteq C \wedge C(o)\}$  (and by construction of the ontologies,  $C_a^{\text{sup}}$  and  $C_a^{\text{supO}}$  are unique) and  $\{C_a \supset C_b^{\text{sub}}\} = \{C_a \supset C \mid C \in \mathcal{O}_b \wedge C \subseteq C_b \wedge C(o)\}$ . If the initial correspondence of the alignment is an equivalence-relation, i.e. if  $\langle C_a, C_b, \supset \rangle \in A_{ab}$ , then the corresponding dynamic upgrade for delete is  $\uparrow(C_a \not\supset C_b \wedge C_a \not\supset C_b)$ . The upgrades for the other adaptation operators remain the same.

Again, as was the case for the adaptation operators on ARG states,  $\delta(\text{add})$ ,  $\delta(\text{addjoin})$ ,  $\delta(\text{refine})$  and  $\delta(\text{refadd})$  entail  $\delta(\text{replace})$ , and  $\delta(\text{refadd})$  entails  $\delta(\text{addjoin})$  and  $\delta(\text{refine})$ .

**Example 5.6 (Running Example - Success).** When ARG is played with  $\blacktriangle$ , agent  $b$  announces that  $!Small_b(\blacktriangle)$  and the correspondence used is  $\langle Black_a, Small_b, \supset \rangle \in A_{ab}$ . This information is compatible with the information of agent  $a$ :  $Black_a$  is compatible with  $Small_a \sqcap Black_a$ , i.e. the most specific class of  $\blacktriangle$ .

Compared to ARG where the round is now finished, there are additional epistemic-doxastic changes on the corresponding DEOL

model. The announcement carries more information than just indicating that the round of ARG was a success, it *transforms* some beliefs of agent  $a$  into knowledge:  $B_a(Small_b(\blacktriangle))$  becomes  $K_a(Small_b(\blacktriangle))$ . In other words, agent  $a$  is now given concrete evidence that  $\blacktriangle$  is a member of  $Small_b$  whereas before she only believed this. Figure 4 can be compared to and the upper schema of Figure 5 for an overview of the changes to the epistemic-doxastic state of agent  $a$ .



**Figure 5: The knowledge (solid black) and belief (dashed red) of agent  $a$  of Example 3.3 after the announcement  $!Small_b(\Delta)$  (above) and after the announcement  $!Small_b(\blacktriangle)$  (below).**

**Example 5.7 (Running Example - Failure).** If instead ARG is played with  $\Delta$ , the round is a failure. Agent  $b$  announces  $!Small_b(\Delta)$  using the same correspondence  $\langle Black_a, Small_b, \supset \rangle \in A_{ab}$ . However, this information contradicts the knowledge of agent  $a$  and, as a result, the correspondence (belief) of the alignment will be dropped.

However, this is not the only revised belief. The contradicted initial beliefs turn into knowledge of their negation. For example,  $B_a(\neg Small_b(\Delta))$  becomes  $K_a(Small_b(\Delta))$  after the announcement. Compare also Figure 4 and the lower schema of Figure 5 for an overview of the changes to the epistemic-doxastic state of agent  $a$ .

According to ARG, an adaptation operator is applied, which results in an updated alignment as explained in Example 3.5. These correspondences should be amongst the beliefs of the agents at the end of the round of ARG. However, for some operators, the correspondences are *already* believed by agent  $a$  before the adaptation operator is applied. We will see why in the next section.

The translation provided in this section is faithful because the semantics of ontologies and alignments we use is the same as in Description Logic and the only dynamic epistemic component arises

from modeling the agents' knowledge and beliefs, see also [4]. A formal proof is out of the scope of this paper.

## 6 FORMAL PROPERTIES OF THE ADAPTATION OPERATORS

With the formal representation of ARG in DEOL we can explore the correctness, redundancy and completeness of the operators. For this, we consider the diagram as pictured in Figure 3.

### 6.1 Correctness

To show that the adaptation operators are correct, we need to show that the diagram of Figure 3 commutes.

**Definition 6.1 (Correctness).** Adaptation operator  $\alpha$  is *correct* if and only if  $\forall s: (z(s))^{\delta(\alpha)} \models z(\alpha(s))$ .

**PROPOSITION 6.2.** The adaptation operators delete, replace, addjoin, refine and refadd are correct.

**PROOF.** We do the proof for agent  $a$  and adaptation operator addjoin. The proof for replace now follows because it is entailed by addjoin, and the proof for refine is symmetric.

Because addjoin only adds beliefs, it suffices to show that these are entailed:  $(z(s))^{!C_b(o); \uparrow(C_a \sqsupset C_b \wedge C_a^{supO} \sqsupset C_b)} \models B_i(C_a \sqsubseteq C_b) \wedge B_i(C_a^{supO} \sqsupset C_b)$ . This holds because initially the correspondence is believed, i.e.  $z(s) \models B_i(C_a \sqsupset C_b)$ , and the upgrade  $!C_b(o); \uparrow(C_a \sqsupset C_b \wedge C_a^{supO} \sqsupset C_b)$  deletes all the worlds from  $z(s)$  in which  $C_b(o)$  is false and then rearranges the remaining worlds such that the ' $C_a \sqsupset C_b \wedge C_a^{supO} \sqsupset C_b$ '-worlds become more plausible than the ' $\neg(C_a \sqsupset C_b \wedge C_a^{supO} \sqsupset C_b)$ '-worlds. Because there remain ' $C_a \sqsupset C_b \wedge C_a^{supO} \sqsupset C_b$ '-worlds accessible for both agents, the belief is enforced. For agent  $b$ , this is true because the announcement  $!C_b(o)$  does not alter her epistemic-doxastic state (she already knew that  $C_b(o)$  as it is in her ontology), and for agent  $a$ , because the announcement  $!C_b(o)$  deletes the worlds in which  $C_a \sqsubseteq C_b$  ( $\neg C_a(o)$  holds because the correspondence and announcement caused a failure) or  $C_a \equiv C_b$  but not those in which  $C_a \sqsupset C_b$  or  $C_a^{supO} \sqsupset C_b$ . Therefore the beliefs  $B_i(C_a \sqsubseteq C_b)$  and  $B_i(C_a^{supO} \sqsupset C_b)$  are enforced for agents  $i \in \{a, b\}$ . Hence addjoin is correct.  $\square$

Yet, the adaptation operator add is not correct because it does not take into account whether the immediate superclass of  $C_a$  is consistent with the object  $o$ . And if it is consistent, add is equivalent to addjoin.

**PROPOSITION 6.3.** The adaptation operator  $\alpha = \text{add}$  is *incorrect*, i.e.  $\exists s: (z(s))^{\delta(\text{add})} \not\models z(\text{add}(s))$ , and  $\forall s$  s.t.  $(z(s))^{\delta(\text{add})} \models z(\text{add}(s))$ :  $\text{add}(s) = \text{addjoin}(s)$ .

**PROOF.** We need to prove the existence of an ARG state  $s$  where  $(z(s))^{\delta(\text{add})} \not\models z(\text{add}(s))$  with upgrade  $\delta(\text{add}) = !C_b(o); \uparrow(C_a \sqsupset C_b \wedge C_a^{sup} \sqsupset C_b)$ , object  $o$  s.t.  $O_b \models C_b(o)$  and  $\langle C_a, C_b, \sqsupset \rangle \in A_{ab}$  the failing correspondence. Pick  $s$  to be any such ARG state where the immediate superclass  $C_a^{sup}$  of  $C_a$  is incompatible with  $o$ , i.e.  $O_a \not\models C_a^{sup}(o)$ . Then  $z(s) \models K_a(\neg C_a^{sup}(o))$  and  $(z(s))^{\delta(\text{add})} \models K_a(C_b(o)) \wedge K_a(C_a^{sup} \sqsupset C_b)$ . This is because  $\delta(\text{add})$  deletes all

' $\neg C_b(o)$ '-worlds from  $z(s)$  and therefore also all the worlds accessible by agent  $a$  where  $C \sqsupset C_b$  for  $C$  such that  $z(s) \models K_a(C(o))$ . In particular, this holds for  $C_a^{sup}$ . But, after applying the adaptation operator add,  $\langle C_a^{sup}, C_b, \sqsupset \rangle$  becomes part of the alignment, so that  $z(\text{add}(s)) \models B_a(C_a^{sup} \sqsupset C_b)$ . Hence  $(z(s))^{\delta(\text{add})} \not\models z(\text{add}(s))$ .

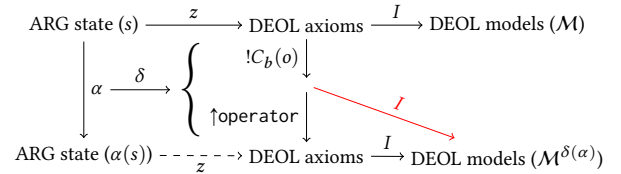
Moreover, whenever  $(z(s))^{\delta(\text{add})} \models z(\text{add}(s))$  it must be that  $O_a \models C_a^{sup}(o)$  so that, per definition,  $C_a^{sup} = C_a^{supO}$ , i.e. add is equivalent to addjoin.  $\square$

Proposition 6.3 is in line with initial predictions and experimental results by Euzenat [13, 14]: addjoin shows faster convergence than add. This is because add can force false correspondences to be added to the alignment that can later cause a failure. From these results, it is clear that for a logical agent, add should be abandoned.

### 6.2 Redundancy

The redundancy of some operators in the running example is not a coincidence. For logical agents, i.e. DEOL agents, some adaptation operators are redundant for *every* ARG state: delete, replace and addjoin are redundant with respect to agent  $a$  and refine is redundant with respect to agent  $b$ . Before we define this redundancy with respect to one agent (*partial redundancy*), let us first consider what it means for an operator to be redundant (with respect to *both* agents). An adaptation operator  $\alpha$  is redundant if and only if solely applying  $!C_b(o)$  on the DEOL translation of  $s$  is already sufficient to obtain an interpretation of the DEOL translation of  $\alpha(s)$ .

**Definition 6.4 (Redundancy).** Adaptation operator  $\alpha$  is *redundant* if and only if  $\forall s: (z(s))^{!C_b(o)} \models z(\alpha(s))$ .



**Figure 6: Diagram of translations between ARG states DEOL axioms that are interpreted on DEOL models, adaptation operators and dynamic upgrades as in Figure 3 where the operators are redundant.**

The adaptation operators discussed here are not redundant, but *partially redundant*. This means that they are redundant with respect to one agent. To prove redundancy, we show that the knowledge and belief of this agent are invariant to the application of the adaptation operator. In fact, because adaptation operators only alter the beliefs of agents, it suffices to show partial redundancy by showing that the beliefs of that agent remain unchanged.

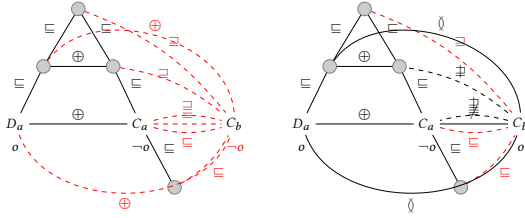
**Definition 6.5 (Partial Redundancy).** An adaptation operator  $\alpha$  is *partially redundant* for agent  $a$  if and only if  $(z(s))^{!C_b(o)} \models B_a\phi$  implies  $z(\alpha(s)) \models B_a\phi$  for each ARG state  $s$  and each  $\phi$  in  $\mathcal{L}_{DEOL}$ .

**PROPOSITION 6.6.** The adaptation operators delete, replace and addjoin are *partially redundant* with respect to agent  $a$ , and refine is *partially redundant* with respect to agent  $b$ .



PROOF. We do the proof for agent  $a$  and the adaptation operator *addjoin*. The proof for *replace* now follows because it is entailed by *addjoin*, and the proof for *refine* is symmetric.

Thus we need to show that  $(z(s))^{!C_b(o)} \models B_a\phi$  implies that  $z(\text{addjoin}_{\langle C_a, C_b, \supseteq \rangle(o)}(s)) \models B_a\phi$ . So consider a sentence  $\phi$  that is *not* believed by agent  $a$  in  $z(\text{addjoin}_{\langle C_a, C_b, \supseteq \rangle(o)}(s))$ , but is in  $z(s)$ . By construction of the dynamics of the operator *addjoin*, this can only be (1) a belief that is inconsistent with  $C_b(o)$  (because the announcement  $!C_b(o)$  deletes these worlds), or (2)  $C_a \not\sqsubseteq C_b$  (because it is enforced by the conservative upgrade part of the dynamics). But these are also not believed by agent  $a$  in  $(z(s))^{!C_b(o)}$ : (1) because  $!C_b(o)$  has deleted all these beliefs, and (2) because  $z(s) \models K_a(\neg(C_a(o)))$  and this knowledge is invariant under the announcement  $!C_b(o)$ , causing the belief in  $C_a \not\sqsubseteq C_b$  to be dropped. Hence, by contraposition, *addjoin* is partially redundant with respect to agent  $a$ . In Figure 7 the knowledge and belief of agent  $a$  is illustrated before and after the announcement  $!C_b(o)$  for an intuition.  $\square$



**Figure 7: The knowledge (solid black) and beliefs (dashed red) of agent  $a$  before (left) and after (right) the announcement  $!C_b(o)$ .**

However, none of the adaptation operators are redundant with respect to both agents. Even the simple delete carries valuable information to agent  $b$ : namely that the initial correspondence fails. Without this operator, agent  $b$  would not be aware whether the round of ARG is a success or a failure.

### 6.3 Incompleteness

Finally, we consider completeness of the adaptation operators: do the operators capture all the information that can be learned? Intuitively, this is proven by comparing what is learned by the agents in ARG scenarios from application of the adaptation operators with what is learned by logical agents in DEOL from the dynamic upgrades. If the former implies the later, the operator is (epistemically) complete.

**Definition 6.7 (Completeness).** Adaptation operator  $\alpha$  is complete if and only if  $\forall s: z(\alpha(s)) \models (z(s))^{\delta(\alpha)}$ .

**PROPOSITION 6.8.** All adaptation operators (delete, replace, add, addjoin, refine, refadd) are incomplete.

PROOF. Again, consider the knowledge and belief of agent  $a$  before and after the announcement  $!C_b(o)$ , see also Figure 7. After the announcement  $!C_b(o)$ , agent  $a$  receives concrete information that object  $o$  belongs to the class  $C_b$ , i.e. she comes to *know* this

information:  $(z(s))^{!C_b(o)} \models K_a(C_b(o))$ . And, by definition, this knowledge remains after application of *any* conservative upgrade, i.e.  $(z(s))^{\delta(\alpha)} \models K_a(C_b(o))$ . Yet, this knowledge is never acquired through application of the adaptation operators because they only concern the alignment, i.e. beliefs of class relations, and not knowledge of instance classification. Hence  $z(\alpha(s)) \not\models K_a(C_b(o))$  and  $z(\alpha(s)) \not\models (z(s))^{\delta}$ .  $\square$

The incompleteness proof of the adaptation operators relies on the agent not memorizing the failure of the correspondence with the drawn object. Yet, from Figure 7 it is clear that there is more knowledge gained by the agents from the announcement  $!C_b(o)$ . This occurs because we measure completeness, and correctness, with respect to the full knowledge and belief of the agent. When concentrating on the alignment only, as expressed by adaptive agents, the operators may be complete.

## 7 DISCUSSION AND CONCLUSION

We developed a theoretical framework for knowledge and belief evolution in ARG and formally defined correctness, completeness and redundancy of adaptation operators to compare adaptive agents and logical agents. We complement the current experimental approach by proving that, in this framework, all but the add operator are correct, delete, replace, addjoin and refine are redundant for one agent and that all operators are incomplete. This contributes to theoretically comparing the different operators and could inspire new adaptation operators for ARG. However, this does not mean that the adaptive agents are meaningless. In fact, the adaptive agents in [11, 13] implement deliberately a ‘sublogical’ behavior and the experiments show that, in some cases, they can perform well. For instance, agents do not need to be fully complete, and not even fully correct, to reach 100% success in ARG. The purpose of our work is to examine them under a logical light. We compare adaptive agents to logical agents, and we prove the logical limitations of the adaptation operators.

Yet, the scope of this paper lays beyond ARG. We have provided a theoretical framework that can be extended to establish formal properties of other games that are designed for agents to improve and repair alignments through interaction. This allows for a theoretical comparison of different dynamic matching algorithms.

In this paper, public signature awareness was a prerequisite in the translation from ARG states to DEOL to capture the dynamics of the game by announcements and conservative upgrades. In the future, we want to drop this prerequisite. We suspect that this might provide the means to capture the ability to generate new (random) correspondences [13].

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